Rev Q1

1. (i) $f(1) = -5 < 0 \quad f(2) = 8 > 0 \quad \Rightarrow \cdots$

(ii) $x^5 = 3x^3 - x^2 + 4. \quad x^3 = 3x - 1 + \frac{4}{x^2} \quad \Rightarrow \cdots$

(iii) 1.78

2. (i) $dx = \sqrt{3}\sec^2\theta\, d\theta \quad x = 0 \quad \theta = 0, \quad x = 1. \quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$$I = \int_0^{\frac{\pi}{6}} \frac{9}{9\sec^4\theta} \sqrt{3}\sec^2\theta\, d\theta = \int_0^{\frac{\pi}{6}} \sqrt{3}\cos^2\theta\, d\theta = \sqrt{3}\int_0^{\frac{\pi}{6}} \cos^2\theta\, d\theta.$$

(ii) $I = \frac{\sqrt{3}}{2}\int_0^{\frac{\pi}{6}} (\cos 2\theta + 1)\, d\theta = \frac{\sqrt{3}}{2}\left(\frac{\sin 2\theta}{2} + \frac{\theta}{2}\right)_0^{\frac{\pi}{6}} = \frac{\sqrt{3}}{2}\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\pi}{12}\right) - 0 = \frac{3}{8} + \frac{\sqrt{3}\pi}{24}.$

3. (i) $\cos y \cdot y' \cdot \ln x + \sin y \cdot \frac{1}{x} = 1 - 2\cos y \cdot y'.$

$\therefore y' \cdot \cos y \cdot (\ln x + 2) = 1 - \frac{\sin y}{x}. \quad y' = \frac{x - \sin y}{x\cos y(\ln x + 2)}$

(ii) $y' = 0 \Rightarrow x = \sin y. \Rightarrow \quad x\ln x = x - 2x. \quad \Rightarrow x\ln x = -x.$

$x \neq 0$ ($\ln x$ is defined for $x > 0$). $\Rightarrow \ln x = -1. \quad x = \frac{1}{e}.$

4. (i) $\int e^{-y}\, dy = \int xe^x\, dx \quad \therefore -e^{-y} = xe^x - e^x + c.$

$x = 0.\ y = 0: \quad -1 = -1 + c \quad c = 0 \quad \therefore -e^{-y} = xe^x - e^x.$

$\therefore e^{-y} = e^x(1-x). \quad y = -\ln[e^x(1-x)] = -x - \ln(1-x).$

(ii) $1 - x = \frac{e^{-y}}{e^x} > 0 \quad \therefore x < 1.$

5 (i) $A + \frac{B}{x} + \frac{Cx + D}{x^2+2} = \frac{3x^3 + 6x - 8}{x(x^2+2)} = 3 + \frac{-8}{x(x^2+2)} \Rightarrow -8 = B(x^2+2) + (Cx+D)\cdot x$

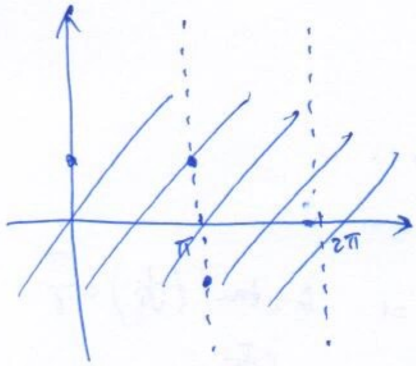
$\Rightarrow B = -4.\ C = 4.\ D = 0 \quad \Rightarrow f(x) = 3 - \frac{4}{x} + \frac{4x}{x^2+2}.$

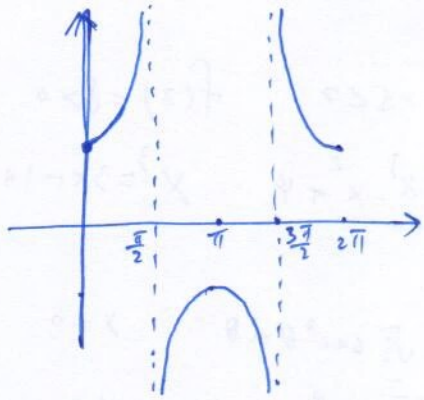
(ii) $\int_1^2 f(x)\, dx = \left[3x - 4\ln x + 2\ln(x^2+2)\right]_1^2 = (6 - 4\ln 2 + 2\ln 6) - (3 - 0 + 2\ln 3)$

$= 3 - 4\ln 2 + 2\ln 2 = 3 - \ln 4.$

RevQ2

1.





2 (i) $y - \frac{1}{y} = 1$. $y^2 - y - 1 = 0$

(ii). $y = \frac{1 \pm \sqrt{5}}{2}$ $\because y > 0$ $\therefore y = \frac{1+\sqrt{5}}{2}$ $\therefore x = \log_2 y = \log_2\left(\frac{1+\sqrt{5}}{2}\right) \doteq 0.694$

3 (i). $LHS = \frac{1}{4}\sin^2 2\theta = \frac{1}{4} \cdot \frac{1}{2}(1 - \cos 4\theta) = RHS$.

(ii) $\int_0^{\frac{\pi}{3}} \sin^2\theta \cos^2\theta \, d\theta = \frac{1}{8}\int_0^{\frac{\pi}{3}} (1 - \cos 4\theta) d\theta = \frac{1}{8}\left(\theta - \frac{1}{4}\sin 4\theta\right)_0^{\frac{\pi}{3}} = \frac{\pi}{24} - \frac{1}{32}\cdot\left(-\frac{\sqrt{3}}{2}\right) - 0$

$= \frac{\pi}{24} + \frac{\sqrt{3}}{64}$.

4. $\int \frac{y^2}{y^3+1} dy = \int dx$. $\frac{1}{3}\ln|y^3+1| = x + C$.

$x = 0.\ y = 1$: $\frac{1}{3}\ln 2 = C$. $\Rightarrow$ $\frac{1}{3}\ln(y^3+1) = x + \frac{1}{3}\ln 2$.

$y^3 + 1 = e^{3x} \cdot 2$ $y = \sqrt[3]{2 \cdot e^{3x} - 1}$.

5. (i). $A(1.0)$

(ii) $y' = \frac{\frac{1}{x}\cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{1 - 2\ln x}{x^3}$. $y' = 0 \Rightarrow x = e^{\frac{1}{2}} \Rightarrow M\left(\sqrt{e}, \frac{1}{2e}\right)$

(iii) $\int_1^e \frac{\ln x}{x^2} dx = \left[\left(-\frac{1}{x}\right)\cdot \ln x\right]_1^e - \int_1^e \left(-\frac{1}{x}\right)\cdot\frac{1}{x}dx = \left(-\frac{1}{e} - 0\right) + \left(-\frac{1}{x}\right)_1^e = 1 - \frac{2}{e}$.

Rev Q3

1. $(1+4x)^{-\frac{1}{2}} = 1 - \frac{1}{2}\cdot 4x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{1\cdot 2}(4x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1\cdot 2\cdot 3}(4x)^3 + \cdots$

$= 1 - 2x + 6x^2 - 20x^3 + \cdots$

2. (i). $dx = \sec^2\theta\, d\theta$

LHS $= \int \frac{1-\tan^2\theta}{\sec^4\theta}\cdot \sec^2\theta\, d\theta = \int (\cos^2\theta - \sin^2\theta)\, d\theta$ = RHS.

(ii) $\cdots = \int_0^{\frac{\pi}{4}} \cos 2\theta\, d\theta = \left(\frac{1}{2}\sin 2\theta\right)_0^{\frac{\pi}{4}} = \frac{1}{2} - 0 = \frac{1}{2}.$

3. (i) $\int \frac{1}{y(4-y)}\, dy = \int \left(\frac{\frac{1}{4}}{y} + \frac{\frac{1}{4}}{4-y}\right) dy = \frac{1}{4}\ln\left|\frac{y}{4-y}\right| + c.$

(ii) $\int \frac{1}{y(4-y)}\, dy = \int dx$ $\frac{1}{4}\ln\left|\frac{y}{4-y}\right| = x + c$

$x=0$ $y=1$: $\frac{1}{4}\ln\frac{1}{3} = c \Rightarrow \ln\left(\frac{y}{4-y}\right) = 4x + \ln\frac{1}{3}.$

$\frac{y}{4-y} = e^{4x}\cdot\frac{1}{3}. \Rightarrow \cancel{y = \frac{4e^{4x}}{4e^4+12}}$ $y = \frac{4e^{4x}}{e^{4x}+3}.$

(iii) $x \to +\infty$. $e^{-4x} \to 0$. $y = \frac{4}{1+3e^{-4x}} \to \frac{4}{1} = 4.$

4 (i) $y' = \frac{1\cdot(x^2+1) - x\cdot(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$ $y'=0$ $x=1$. $(x>0)$

(ii) $\int_0^p \frac{x}{x^2+1}\, dx = \frac{1}{2}\ln(x^2+1)\Big|_0^p = \frac{1}{2}\ln(p^2+1)$

(iii). $\frac{1}{2}\ln(p^2+1) = 1.$ $p^2+1 = e^2$ $p = \sqrt{e^2-1} = 2.53.$

Rev Q4

1. $\int_0^1 x e^{2x}\,dx = \left(x\cdot\frac{1}{2}e^{2x}\right)_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx = \frac{1}{2}e^2 - 0 - \left(\frac{1}{4}e^{2x}\right)_0^1$

$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4}(e^2+1).$

2. line AB: $\vec{r} = \begin{pmatrix}2\\2\\1\end{pmatrix} + t\begin{pmatrix}-1\\2\\2\end{pmatrix}.$ $\begin{cases} 2-t = 4+s & ① \\ 2+2t = -2+2s & ② \\ 1+2t = 2+s & ③ \end{cases}$

$\left\{\begin{array}{l} ①, ② \Rightarrow \quad s=0,\ t=-2. \\ \text{in } ③: \quad 1+2t=-3. \quad 2+s=2 \qquad LHS \neq RHS \Rightarrow \ldots \end{array}\right.$

3.(i) $f(x) = \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ $\quad A(x-2)^2 + B(x-2)(2x+1) + C(2x+1) = 9x^2+4$

$x=2$: $C=8$; $x=-\frac{1}{2}$: $A=1$; $x=0$. $4A-2B+C=4$. $B=4$.

$\therefore f(x) = \frac{1}{2x+1} + \frac{4}{x-2} + \frac{8}{(x-2)^2}.$

(ii). $f(x) = (1+2x)^{-1} - 2\left(1-\frac{1}{2}x\right)^{-1} + 2\left(1-\frac{1}{2}x\right)^{-2}$

$= (1-2x+4x^2+\cdots) - 2\left(1+\frac{1}{2}x+\frac{1}{4}x^2+\cdots\right) + 2\left(1+x+3\cdot\frac{x^2}{4}+\cdots\right)$

$= 1+x+5x^2+\cdots$

4 (i). $\frac{dx}{dt} = k\cdot(100-y).$ $\quad x=5$: $\frac{dx}{dt}=1.9$ $\quad\therefore 1.9 = k\cdot 95.$ $\quad k=0.02 \Rightarrow \ldots$

(ii) $\int \frac{50}{100-x}dx = \int dt$; $\quad -50\ln|100-x| = t+c$, $\quad t=0,\ x=5$: $-50\ln 95 = c$

$\therefore \ln(100-x) = -\frac{t}{50} + \ln 95.$ $\quad 100-x = 95\cdot e^{-\frac{t}{50}}$ $\quad x = 100-95e^{-\frac{t}{50}}.$

(iii) $t\to+\infty$. $e^{-\frac{t}{50}}\to 0$ $\quad x\to 100.$

5 (i) $y' = \frac{1}{x} - \frac{2}{x^2} = \frac{x-2}{x^2}$. $\quad y'=0 \Rightarrow x=2.$ $\quad (2, \ln 2+1)$

$y'' = -\frac{1}{x^2} + \frac{4}{x^3}$ $\quad x=2$: $y'' = -\frac{1}{4}+\frac{1}{2} > 0 \Rightarrow$ minimum point.

(ii) $\alpha = \frac{2}{3-\ln\alpha}$. $\Rightarrow 3-\ln\alpha = \frac{2}{\alpha}$, $\quad 3 = \ln\alpha + \frac{2}{\alpha} \Rightarrow \ldots$

(iii). $\alpha = 0.56$

Rev Q5

1. $\log_3 x = \log_3 4 + (-y)$. $\quad y = \log_3 4 - \log_3 x$.

2. when $x \geq 1$: $\quad 2x > x-1 \quad x > -1 \quad \therefore x \geq 1$
when $x < 1$: $\quad 2x > 1-x. \quad x > \frac{1}{3} \quad \therefore \frac{1}{3} < x < 1$
$\Rightarrow x > \frac{1}{3}$.

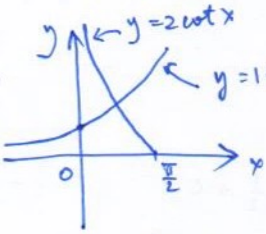
3. $\dfrac{dy}{dx} = \dfrac{2\sin 2\theta}{2 + 2\cos 2\theta} = \dfrac{4\sin\theta\cos\theta}{2 + 4\cos^2\theta - 2} = \tan\theta$.

4 (i) $7\cos\theta + 24\sin\theta = 25\cos(\theta - 73.74°)$. $\quad \left(\alpha = \tan^{-1}\frac{24}{7} = \cdots\right)$

(ii) $25\cos(\theta - 73.74°) = 15$. $\quad \theta - 73.74° = \pm\cos^{-1}\frac{3}{5} + 360°\cdot k = \pm 53.13° + 360°\cdot k$.
$\therefore \theta_1 = 73.74° + 53.13° = 126.9°$. $\quad \theta_2 = 73.74° - 53.13° = 20.6°$.

5. (i) $\dfrac{dx}{dt} = kx - 25$. $\quad x = 1000, \frac{dx}{dt} = 75$: $\quad 75 = 1000k - 25$. $\quad k = 0.1 \Rightarrow \cdots$

(ii) $\displaystyle\int \frac{dx}{x-250} = \int 0.1\, dt$. $\quad \ln|x - 250| = 0.1t + c$. $\quad t = 0, x = 1000$: $\quad c = \ln 750$
$\therefore x - 250 = 750 \cdot e^{0.1t} \quad x = 750e^{0.1t} + 250$.

6 (i).


(ii) $f(x) = 2\cot x - 1 - e^x$. $\Rightarrow \cdots$
$f(0.5) = 1.01 > 0$. $\quad f(1) = -2.15 > 0$

(iii) $\dfrac{2}{1+e^x} = \tan x \quad \Rightarrow \cdots$ $\qquad$ (iv) 0.61.

7. (i). $\vec{r} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$.

(ii). $\begin{pmatrix} -1+3t-3 \\ 3-t+1 \\ 5-4t+4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix} = 0 \quad \Rightarrow \quad 9t - 12 + t - 4 + 16t - 36 = 0 \quad t = 2$.
$\therefore N: \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$. $\quad |\overrightarrow{BN}| = \left|\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}\right| = 3$.

8. (i) $y' = \frac{1}{2}x^{-\frac{1}{2}}\ln x + x^{\frac{1}{2}} \cdot \frac{1}{x} = \frac{1}{\sqrt{x}}\left(\frac{\ln x}{2} + 1\right) \quad y' = 0 \Rightarrow \ln x = -2 \quad x = e^{-2}$.

(ii) $\displaystyle\int_1^4 \sqrt{x}\ln x\, dx = \left(\frac{2}{3}x^{\frac{3}{2}} \cdot \ln x\right)_1^4 - \int_1^4 \frac{2}{3}x^{\frac{1}{2}} dx = \frac{2}{3}\cdot 8 \cdot \ln 4 - 0 - \left(\frac{4}{9}x^{\frac{3}{2}}\right)_1^4 = \frac{16}{3}\ln 4 - \frac{4}{9}\cdot 7 = 4.28$.

9 (i) $\dfrac{10}{(2-x)(1+x^2)} = \dfrac{A}{2-x} + \dfrac{Bx+C}{1+x^2} \qquad 10 = A(1+x^2) + (Bx+C)(2-x). \quad A = 2. \quad B = 2. \quad C = 4$.
$\therefore \cdots = \dfrac{2}{2-x} + \dfrac{2x+4}{1+x^2}$.

(ii). $\cdots = \left(1 - \frac{x}{2}\right)^{-1} + (2x+4)(1+x^2)^{-1} = \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + \cdots\right) + (2x+4)(1 - x^2 + \cdots)$
$= 5 + \frac{5}{2}x - \frac{15}{4}x^2 - \frac{15}{8}x^3 + \cdots$

Rev Q6

1. $(2+3x)^{-2} = \frac{1}{4}(1+\frac{3}{2}x)^{-2} = \frac{1}{4}(1-3x+3\cdot\frac{9}{4}x^2) = \frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 + \cdots$

2. (i) $p(-2)=0 \Rightarrow -8+4+a=0. \quad a=4$

(ii) $x^3-2x+4=(x+2)(x^2-2x+2) \quad \Rightarrow \quad x^2-2x+2.$

3. $y' = 1\cdot\sin 2x + x\cdot\cos 2x\cdot 2 \quad x=\frac{\pi}{4}: \; y'=1 \quad y=\frac{\pi}{4}. \Rightarrow \cdots$ is $y-\frac{\pi}{4}=x-\frac{\pi}{4}$ or $y=x$.

4. $u=2+\frac{1}{u}. \quad u^2-2u-1=0. \quad u=1\pm\sqrt{2}. \quad u>0 \Rightarrow u=1+\sqrt{2} \quad x=\log_3(1+\sqrt{2})=0.802$

5. (i) $2\cos(\theta-\frac{\pi}{3})$.

(ii) $\cdots = \int_0^{\frac{\pi}{2}} \frac{d\theta}{4\cos^2(\theta-\frac{\pi}{3})} = \left[\frac{1}{4}\tan(\theta-\frac{\pi}{3})\right]_0^{\frac{\pi}{2}} = \frac{1}{4}\tan\frac{\pi}{6} - \frac{1}{4}\tan(-\frac{\pi}{3}) = \frac{1}{4}\cdot\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{4} = \frac{1}{\sqrt{3}}.$

6. (i) $\frac{1}{2}r^2\sin\alpha = \frac{1}{2}\cdot\frac{1}{2}r^2\alpha \quad \Rightarrow \quad 2\sin\alpha=\alpha$

(ii) $f(x)=x-2\sin x. \quad f(\frac{\pi}{2})=\frac{\pi}{2}-2<0. \quad f(\frac{\pi}{3})=\frac{\pi}{3}-\sqrt{3}>0 \Rightarrow \cdots$

(iii) $x=\frac{1}{3}(x+4\sin x). \quad \Rightarrow \quad 3x=x+4\sin x \quad \Rightarrow \quad x=2\sin x.$

(iv) 1.90

7. (i) $f(x)=1+\frac{-x}{(x+1)(x+3)} = 1+\frac{A}{x+1}+\frac{B}{x+3} \Rightarrow A=\frac{1}{2}. \; B=-\frac{3}{2} \quad \therefore f(x)=1+\frac{\frac{1}{2}}{x+1}-\frac{\frac{3}{2}}{x+3}.$

(ii) $\cdots = \int_0^3 (1+\frac{\frac{1}{2}}{x+1}-\frac{\frac{3}{2}}{x+3})dx = \left[x+\frac{1}{2}\ln(x+1)-\frac{3}{2}\ln(x+3)\right]_0^3 = (3+\frac{1}{2}\ln 4-\frac{3}{2}\ln 6)-(-\frac{3}{2}\ln 3) = 3-\frac{1}{2}\ln 2$

8 (i) $\tan x = S_{\triangle PTN} = \frac{1}{2}\cdot TN\cdot PN = \frac{1}{2}\cdot\frac{PN}{\frac{dy}{dx}}\cdot PN = \frac{1}{2}\frac{y^2}{\frac{dy}{dx}} \quad \Rightarrow \cdots$

(ii) $\int \frac{1}{y^2}dy = \int \frac{1}{2}\cot x\,dx \quad -\frac{1}{y}=\frac{1}{2}\ln(\sin x)+c. \quad -\frac{1}{2}=\frac{1}{2}\ln\frac{1}{2}+c. \quad c=-\frac{1}{2}+\frac{1}{2}\ln 2.$

$\therefore -\frac{1}{y}=\frac{1}{2}\ln(\sin x)-\frac{1}{2}+\frac{1}{2}\ln 2. \quad y=\frac{-1}{\frac{1}{2}\ln(\sin x)-\frac{1}{2}+\frac{1}{2}\ln 2} = \frac{2}{1-\ln(2\sin x)}.$

9 (i) $y' = -\frac{1}{2}e^{-\frac{1}{2}x}\sqrt{1+2x} + e^{-\frac{1}{2}x}\cdot\frac{1}{2}(1+2x)^{-\frac{1}{2}}\cdot 2 = \frac{e^{-\frac{1}{2}x}}{2\sqrt{1+2x}}(-1-2x+2) \quad y'=0: \; x=\frac{1}{2}.$

(ii). $\int_{-\frac{1}{2}}^0 \pi e^{-x}(1+2x)dx = \left[\pi(1+2x)(-e^{-x})\right]_{-\frac{1}{2}}^0 - \int_{-\frac{1}{2}}^0 \pi(-e^{-x})\cdot 2dx$

$= (-\pi-0)+2\pi(-e^{-x})_{-\frac{1}{2}}^0 = -\pi+2\pi(-1+e^{\frac{1}{2}}) = 2\pi\sqrt{e}-3\pi.$

10 (i) $\begin{cases} 1-2t=1+s \\ 5+t=2-s \\ 2-t=3 \end{cases} \Rightarrow t=3. \; s=-6 \quad \Rightarrow \cdots$

$-1\neq 3$

(ii) $\vec{AP}\cdot\vec{AB}=|\vec{AP}|\,|\vec{AB}|\cdot\cos 60° \Rightarrow \left|\begin{pmatrix}-2t\\3+t\\-1-t\end{pmatrix}\right|\cdot\left|\begin{pmatrix}1\\-1\\0\end{pmatrix}\right|\cdot\frac{1}{2} = -2t-3-t$

$\Rightarrow \sqrt{4t^2+(t+3)^2+(t+1)^2}\cdot\sqrt{2}\cdot\frac{1}{2} = -3t-3. \Rightarrow 4t^2+t^2+6t+9+t^2+2t+1=2(3t+3)^2$

$\Rightarrow 12t^2+28t+8=0, \quad 3t^2+7t+2=0.$

(iii) $t=-2$ or $-\frac{1}{3}$. since $-3t-3>0 \;\therefore\; t<-1 \;\therefore\; t=-2.$